THERMODYNAMICS OF SUPERCONDUCTORS

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According to Meissner effect the transition between the normal and superconducting state is thermodynamically reversible. This is because the superconducting current do not die away with the production of joule heats when the superconductivity is destroyed by the application of magnetic field.

As we know that the free energy density of a metal in the normal state is independently the strength of the applied magnetic field, H_a. But in the superconducting state it raise the free energy density g_s of metal by amount $\frac{\mu_0 H_a^2}{2}$. The critical field H_c is that field strength which is required to raise the free energy of the superconducting state above that of the normal state. So the difference in free energy b/w the normal and the superconducting state in an applied field of strength H_a is given by

$$g_n - g_s(H_a) = \frac{1}{2}\mu_0 (H_c^2 - H_a^2) \quad (1)$$

where

H_c = strength of critical field

Now, The Gibbs free energy is is given by

$$G(T, P) = U - TS + PV \quad (11)$$

where

T = Temperature
P = pressure
S = Entropy
M = magnetic moment
U = Internal energy

The free energy of magnetised body is given by the relation

$$G = U - TS + PV - \mu_0 H_a M. \quad -(11)$$

~~Now, on differentiating w.r.t. T keeping P and V constant~~

$$\cancel{\frac{dG}{dT} = dU - (Tds + S}$$

When the temperature is varied by amount dT (keep P and H_a constant).

Then,

$$dG = dU - Tds - SdT + PdV - \mu_0 H_a dM \quad -(IV)$$

By the first law of thermodynamics

$$dU = Tds - \cancel{PdV} + \cancel{\mu_0 H_a dM}$$

$$Tds - P \cdot dV + \mu_0 H_a dM$$

On putting the value of dU in eqn (IV) we get

$$\cancel{dG = Tds - PdV + \mu_0 H_a dM - Tds - SdT + PdV - \mu_0 H_a dM}$$

$$dG = - SdT$$

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$\therefore \quad S = -\left(\frac{\partial G}{\partial T}\right)_{P, H_a}$

Now The entropy for unit volume is given by

Now, $S = -\left(\frac{dg}{dT}\right)_{P, H_a}$

$$S_n - S_s = \frac{\partial}{\partial T}\left[\frac{1}{2}\mu_0\left(H_c^2 - H_0^2\right)\right]$$

$$= \frac{1}{2}\mu_0 \frac{dH_c}{dT} - 0$$

$$= \frac{1}{2}\mu_0 2H_c \frac{dH_c}{dT}$$

$$\boxed{S_n - S_s = \mu_0 H_c \frac{dH_c}{dT}}$$

as $\frac{dH_c}{dT}$ is decreasing with increase in temperature. So, $\frac{dH_c}{dT}$ is +ve.

So, The entropy of the superconducting state is less than that of the normal state. i.e. Degree of the order in a superconducting state is much greater than that in the normal state.

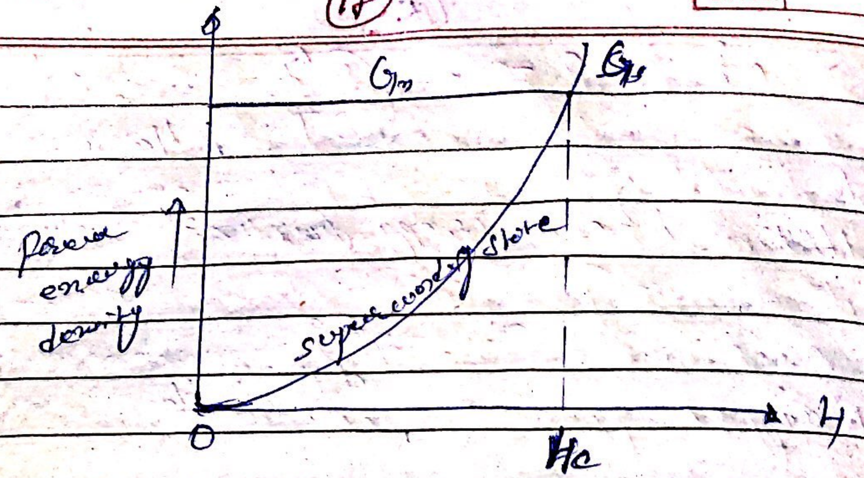

Fig: Free energy versus magnetic field in the normal and superconducting state.

Coherence Length.

The concept of coherence is the idea that superconductivity is due to the mutual interaction and co-relation of the behaviour of electrons which extend over a considerable distance.

The maximum distance upto which the states of pair electrons are correlated to produce superconductivity is called coherence length ξ_0.

The properties of a superconductor depend upon the co-relation of electrons (lie) within a volume of ξ_0^3 called coherence volume.

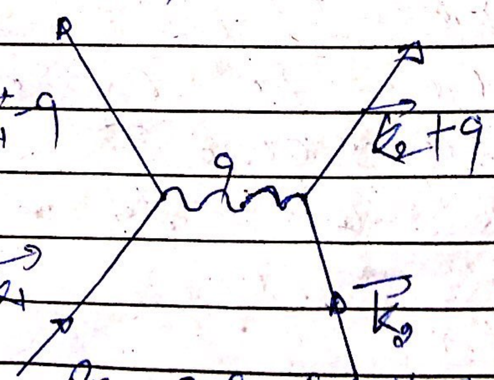

Fig: e-e Interaction through lattice phonon